The “gliding pearl” experiment

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Introduction

The following experiment was developed during the practising time of I.W. at the working place of W.D.B.

Setup and experiment

In order to test out special situations of movement of a mass, we built up a “gliding pearl” experiment. A cylindric mass body with a hole along its axis slide could slide on a round stick filling the hole. The stick itself was fastened perpendicularly to an axis. This axis could be rotated as well.
In the experiment the stick with the body on it were set to a initial position which pointed almost perfectly against gravity. Then the stick was let free and the weight fell down to earth guided by the stick rotating around the axis.
The movement of the body undergoes a “sort of a phase transition”. In the starting phase the body behaves as a rigid body (1 degree of freedom). If a certain angle is reached, the body start to move as well radially (2 degrees of freedom).
The single parts and the whole setup are shown in fig.1. The blueprints of the construction (konstruktion1.cdr in COREL DRAW) can be found in the documenting file collection pearldoc.zip attached to this article.
The measurements has been recorded by a good standard digital video camera using 1/1000 sec recording time.

Exploiting the experimental data

After or before the measurement is recorded it should be done a test measurement which allows to check whether the horizontal and vertical direction have the same scale after the recording and grabbing process. We did this by recording pictures of the stick in horizontal and vertical position. After the videos are grabbed (in TIF) on a video system the pictures, consisting of two half pictures, can be reconstructed fully, if a graphic program is used which has a video interlace filter (for instance PHOTOSHOP). The colour information is discarded, because it makes more easy the data acquisition in the next step.
The acquisition of the data is done using a freeware program ‘NIH image’ to be downloaded at http://rsb.info.nih.gov/nih-image/Default.html
This program allows to combine B/W pictures in a clip (with the “Windows to stack” ! call). Furthermore, it allows to determine the time dependent position of the moving setup in each clip photo by reading off the coordinates of position. After applying the scale correction the radius position and the angle are determined from the coordinate differences using elementary geometric functions like Pythagoras and Arcustangens.

Theory

The solution is done acc. to the standard procedures using the Lagrange equations of second kind as described in the top document.
First the Lagrange energy has to be determined to

$$\mathfrak{L} = T - V = \frac{1}{2}\Theta(t)\,\dot{\varphi}^2 - \left(Mr(t) + m_1(r_0 - \Delta l/2) + m_2 L/2\right) g \sin(\varphi) \qquad \textbf{(1)}$$

with

$$\Theta(t) = M[r^2(t) + (r_i^2 + r_a^2 + \Delta l^2/3)/4] + m_1(r_0 - \Delta l/2)^2 + m_2 L^2/3 \qquad \textbf{(2)}$$

being the time dependent inertial moment Θ. The symbol used can be found in the tab.1 below.

Tab.1:

M	mass of the brass body
r(t)	radius coortdinate of the brass body dependent from time
r_i	inner “mean” radius of the hole in cylindric brass body (we took 5mm !)
r_a	outer radius of the cylindric brass body
r_0	start position of the cylindric brass body
Δl	length of the cylindric brass body
m_1	mass of the stopper
m_2	mass of the stick
g	strenght of the gravitational field

The terms in the eq.1 are from the left: the kinetic energy of the whole setup, the term with the bracket is the potential energy consisting of (from the left) the sum of the potential energy of the brass body, the stopper and the stick .
The terms in eq.2 are from the left: the kinetic energy of the brass body M consisting of the contribution due to the Steiner law and the approximative moment of inertia calculated acc. standard formulas (comp. the file formeln.gif in the in the documenting file collection pearldoc.zip attached to this article) plus the approximative contribution of the stopper (taken as mass point) and the stick.
As long as the centrifugal force on the gliding mass does not overcome the radial force component due to gravitation the system behaves like a rigid body with $r_0 = r(t)$ and the angle position φ being the only independent variable or degree of freedom. Then, at the beginning for the rigid case, the equation of motion can be derived to

$$\Theta\ddot{\varphi} + \left(Mr_0 + m_1(r_0 - \Delta l/2) + m_2 L/2\right) g \cos(\varphi) = 0 \qquad \textbf{(3)}$$

if we take the zero of angle measurement on the right side.
If the radial force overcomes the gravitation meaning if

$$\dot{\varphi}^2 r_0 - g \sin(\varphi) \geq 0 \qquad \textbf{(4)}$$

then we have a system of linear differential equations of second order

$$\Theta(t)\ddot{\varphi} + \left(Mr(t) + m_1(r_0 - \Delta l/2) + m_2 L/2\right) g\cos(\varphi) = 0 \qquad \textbf{(5)}$$

Because the used program MATHEMATICA can solve only linear differential systems numerically the equations are rewritten as system of 2 (for 1 degree of freedom) or 4 (for 2 degree of freedom) equations. For details we refer to the documenting file collection pearldoc.zip attached to this article. The program routines which solve this problem can be found in the files fallex3.nb (for 1 degree of freedom) and fallex4.nb (for 2 degree of freedom) of documentation package attached to this article. The programs can be read with a MATHEMATICA reader to be downloaded at http://www.wolfram.com . The routines are explained in more detail in the file readme.txt of the documenting file collection pearldoc.zip attached to this article.

Results

Angle and angular velocity vs. time are shown in fig. 2 a) . Radius position of the brass body and radial velocity vs. time is shown in fig 2b) .

Discussion

It is clear as well that energy is conserved in this experiment because the system fulfills all conditions of Hamiltonian energy conservation. It is checked as well numerically in the MATHEMATICA - programs attached to this article.

We see that the classical mechanics is able to describe these problem quite exactly.

Figures:

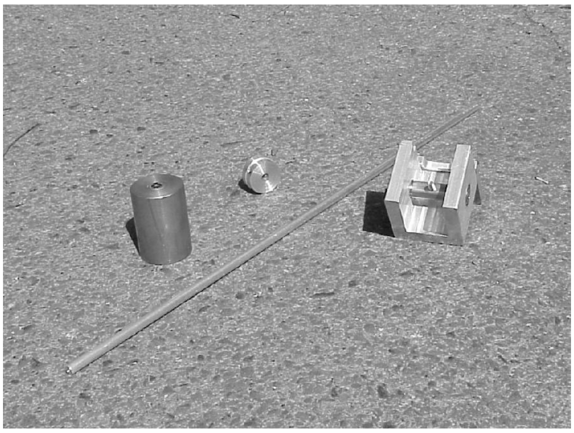

Fig.1a: The single parts of the setup shown in fig.1b .

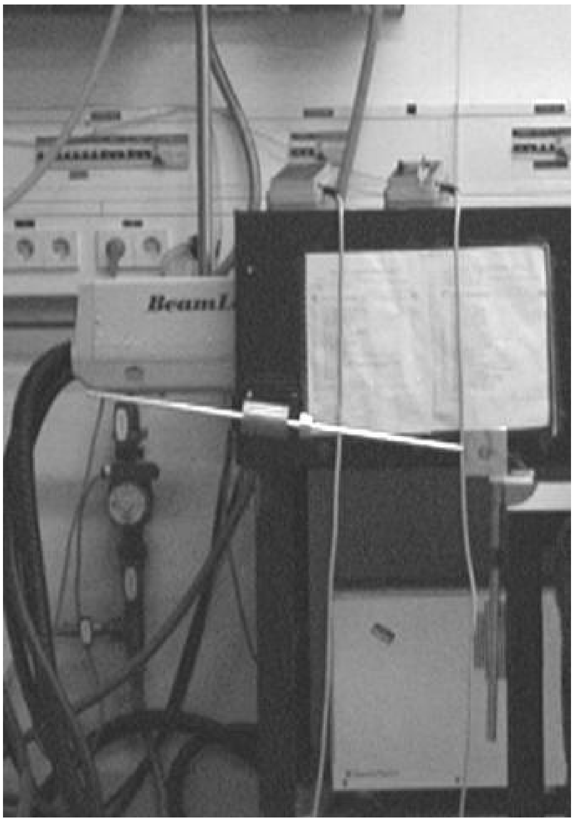

Fig.1b: The gliding pearl experiment picture by a normal video camera with 1/1000 sec recording time

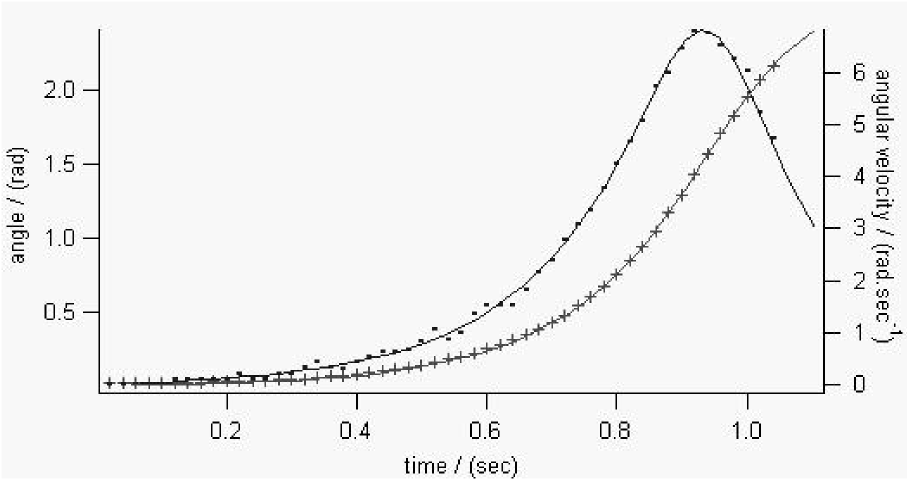

Fig. 2a: angle and angular velocity vs. time for the gliding pearl experiment shown in fig.1 the stick starts at the top position and falls down
angle vs. time are the red curves, the line are theoretical values, markers are experimental values, the scale on the left;
angular velocity vs. time are the blue curves, the line are theoretical values, dots are experimental values, the scale is on the right

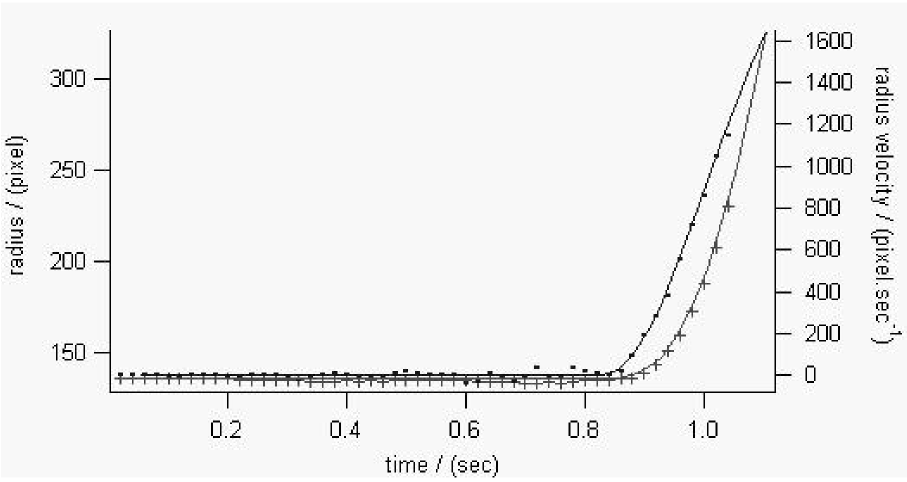

fig. 2b) radius and radial velocity vs. time for the gliding pearl experiment shown in fig.1 the brass body starts at the top position and falls down
radis vs. time are the red curves, the line are theoretical values, markers are experimental values, the scale on the left;
radial velocity vs. time are the blue curves, the line are theoretical values, dots are experimental values, the scale is on the right