The "falling disk" experiment

by W.D. Bauer

Introduction

The following experiment was developed in order to check scope and validity of Wuerth's experimental claims against the theory of mechanics.

Setup and experiment

In order to test out a recent suggest [1] a disk was built into a frame which allow the disk to rotate in it, comp. fig.1 or the blueprints konstruktion2.cdr in the documentation file falldoc.zip.
At one end of the frame was an axis parallel to the axis to the disk. So it was possible to let the disk fall in the gravitational field doing a rotating motion around the axis of the frame. The disk either could move free in the frame either was connected with the frame in order to prevent a own motion of the disk.
A third option was used as well when we build in a brake which braked down the own motion of the disk and changed the setup into a rigid body, comp. fig.2.
The measurements has been recorded by a good standard digital video camera using standard video frequency of 25 Hz and 1/1000 sec recording time.

Exploiting the experimental data

After or before the measurement is recorded it should be done a test measurement which allows to check whether the horizontal and vertical direction have the same scale after the recording and grabbing process. We did this by recording pictures of the stick in horizontal and vertical position. After the videos are grabbed (in TIF) on a video system the pictures, consisting of two half pictures, can be reconstructed fully, if a graphic program is used which has a video interlace filter (for instance PHOTOSHOP). The colour information is discarded, because it makes more easy the data acquisition in the next step.
The acquisition of the data is done using a freeware program 'NIH image' to be downloaded at http://rsb.info.nih.gov/nih-image/Default.html
This program allows to combine B/W pictures in a clip (with the "Windows to stack" ! call). Furthermore, it allows to determine the time dependent position of the moving setup in each clip photo by reading off the coordinates of position. After applying the scale correction the radius position and the angle are determined from the coordinate differences using elementary geometric functions like Pythagoras and Arcustangens.

Theory

The solution is done acc. to the standard procedures using the Lagrange equations of second kind as described in the top document.
First the Lagrange energy has to be determined in the rigid case to

$$\mathfrak{L} = T - V = \frac{1}{2}\left((\Theta_{disk} + \Theta_{frame}) + Mr^2\right)\dot{\varphi}^2 - \left((M+m)r_0\right) g \sin(\varphi) \qquad \textbf{(1)}$$

The symbol used can be found in the tab.1 below.

Tab.1:

M	mass of the disk
Θ_{disk}	inertial moment of the disk
m	mass of the frame
Θ_{frame}	inertial moment of the frame
r	radius coordinate of the centre of gravity of the disk alone
$r_0 \approx r$	radius coordinate of the common centre of gravity of the whole setup
g	strength of the gravitational field
φ	angular coordinate
$\dot{\varphi}$	angular velocity

In the case with the free moving disk we have

$$\mathfrak{L} = T - V = \frac{1}{2}\left((\Theta_{frame} + Mr^2\right)\dot{\varphi}^2 - \left((M+m)r_0\right) g \sin(\varphi) \qquad \textbf{(2)}$$

The terms in the eq.1 and eq.2 are from the left: the kinetic energy of the whole setup consisting of the inertial moment contribution(s) and the contribution due to Steiner's law. The right term is the potential energy consisting of (from the left) the sum of the potential energy of disk and frame .

The numerical computation of this values for the setups presented has been done acc. to standard formulas, comp. the file formeln.gif and readme.txt in the in the documenting file collection falldoc.zip attached to this article.

Because the used program MATHEMATICA can solve only linear differential systems of first order numerically the equation of 2nd order is written as a system of 2 equations of first order (for 1 degree of freedom). For details we refer to the documenting file collection falldoc.zip attached to this article. The program routines which solve this problem can be found in the files FALLSCHEIBEfrei.nb and FALLSCHEIBEstarr.nb in documentation package attached to this article. The programs can be read with a MATHEMATICA reader to be downloaded at http://www.wolfram.com . The routines are explained in more detail in the file readme.txt of the documenting file collection falldoc.zip attached to this article.

The braking process has been modeled acc. to the theory described in section *3.2.A2* of the rotator article including the gravitational potential as accelerating and decelerating force. The equations of motion to be solved are

$$\frac{Mgr_0}{\Theta_1}\cos(\varphi) + \ddot{\varphi} + k(\ddot{\varphi} + \ddot{\alpha}) = 0$$

$$k^*\dot{\alpha} + k(\ddot{\varphi} + \ddot{\alpha}) = 0$$

We refer again to the the file readme.txt in falldoc.zip . The simulation is done by the MATHEMATICA-program FALLSCHEIBEbraked.nb .

Results
Angle and angular velocity vs. time of the setup with the disk free are shown in fig. 2 a) .
Angle and angular velocity vs. time of the setup with the disk rigid are shown in fig. 2 b) .
In both cases experimental values and theoretical simulation coincide.
The braked case is shown as only simulation in fig 3.

Discussion
It is clear as well that energy is conserved in this experiment because the system fulfills all conditions of Hamiltonian energy conservation. It is checked as well numerically in the MATHEMATICA - programs attached to this article.
We see that the classical mechanics is able to describe these problem quite exactly.

Figures:

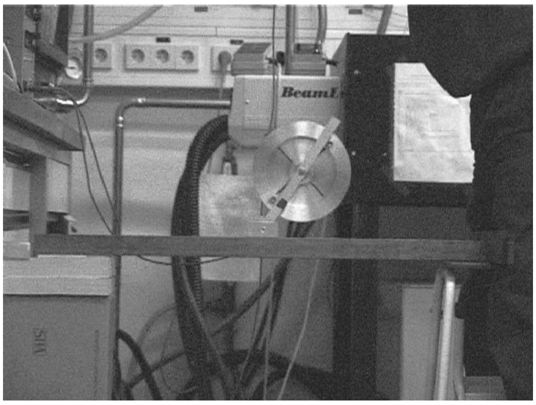

Fig.1a: the falling disk experiment.
1/1000 sec recording time

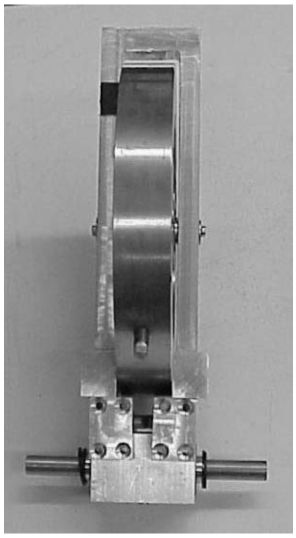

Fig.1b: the disk with a brake built in.
Two iron pieces are sticking in the alu in front and are pressed against each other by springs. If the pin in the disk hits the iron pieces then - due to their form - they are pressed aside against their counter springs in the inner of their alu fitting. Thereby, they let pass the pin. Then, the pin bounces against the frame. If now the disk wants to turn backward , the iron pieces block the movement, the setup is locked and acts as an rigid body.

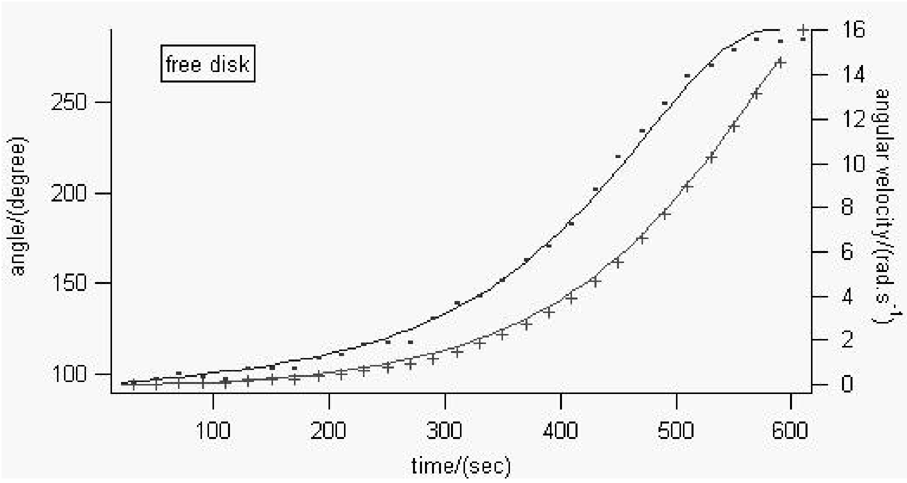

Fig. 2a: angle and angular velocity vs. time for free movable disk in setup 1a)
the disks starts at the top position and falls down
angle vs. time are the red curves, the line are theoretical values, markers are experimental values, the scale is on the left;
angular velocity vs. time are the blue curves, the line are theoretical values, dots are experimental values, the scale is on the right

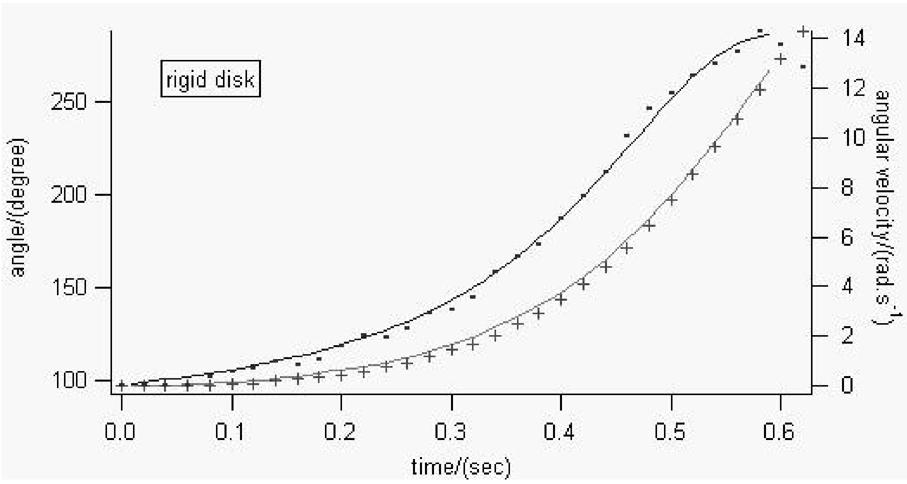

Fig.2b: angle and angular velocity vs. time for rigid non-movable disk in setup 1a)
the disks starts at the top position and falls down
angle vs. time are the red curves, the line are theoretical values, markers are experimental values, the scale on the left;
angular velocity vs. time are the blue curves, the line are theoretical values, dots are experimental values, the scale is on the right

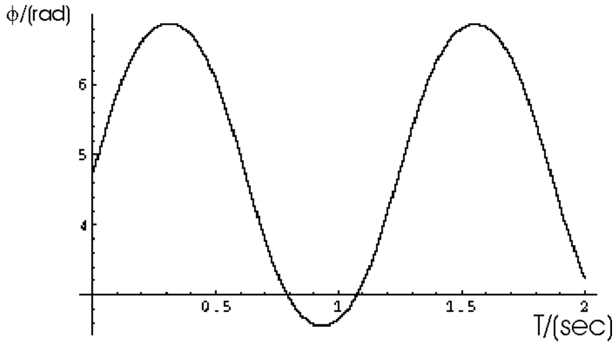

Fig.3: Angle vs. time of the locked disk after the braking process
The diagram is calculated by the MATHEMATICA-program Fallscheibebraked.nb in the documentation package falldoc.zip attached to this article. It is seen that the disk reaches only more than the half of its initial height after the braking process. Afterwards it swings back and forth. This is confirmed in the experiment qualitatively.